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Technical Note

Correlations for natural convection heat transfer in two-layer fluids with internal heat generation

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Abstract

Two simple semiempirical correlations for an estimate of heat transfer in horizontal layers of superposed immiscible fluids with internal heat sources are suggested. Different boundary conditions are considered. The predicted results are compared with the known experimental correlations. The results are of interest to post-accident heat removal in fast and light water reactors.

Keywords: Heat transfer; Natural convection; Analytical and numerical techniques

1. Introduction

Since the last three decades research in nuclear power safety has motivated the study of the buoyancy-driven convection in fluids with volumetric heat generation. Recently, the problem of convective heat transfer in a layered system attracted attention due to possible core melt stratification in a postulated severe accident scenario in a light water reactor [16]. Besides the reactor safety research, thermal convection in a double-layer system is of interest in Chemical Engineering and Geophysics. Much research effort has been directed at the problem of natural convection in a uniform pool with internal heat generation [5]. However, only a few studies have focused on the subject of thermal convection in volumetrically heated stratified layers. Fieg [6] investigated the natural convection characteristics of two stratified immiscible liquid layers with internally heated lower layer. The temperature was maintained equal at the top and bottom boundaries. Heptane and water were used as lighter and heavier liquids, respectively. The important conclusion was that the two layers behaved as if separated by a rigid highly conductive wall. The correlation for the Bénard problem in the upper layer and that for the layer bounded by isothermal walls of different temperatures and volumetrically heated were applied and calculated values agreed with the experimental data to within ±10% accuracy. No heat transfer data, except the measured temperature profile, were presented. Schramm and Reineke [15] studied experimentally and numerically the natural convection in a rectangular channel filled with two immiscible fluids of different physical properties. No heat flux data were obtained in their experiment. In computations isotherms quite similar to those observed in the experiment were obtained. Kulacki and Nguen [13] studied hydrodynamic instability and thermal convection in a horizontal layer of two immiscible fluids with internal heat generation in the lower layer. In their study, the systems of heptane–water or silicone oil–water was bounded in a square cavity from below by a rigid, insulated surface and from above, by an isothermal wall. The heat was generated internally in the lower layer. Experimental measurements of transient and steady state convection up to Rayleigh numbers of 10^{11} were presented. The Nusselt number based on the average heat transfer coefficient for different layer thickness ratios were obtained from the experiments and correlated as a function of the

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Nomenclature

A	dimensionless complex, $A = [(\nu_{12}\alpha_{12})^{n_1}/C_1\beta_{12}^{n_1}k_{12}L_{12}^{3n_1-1}]^{1/(n_1+1)}$
C	constant
c_p	specific heat
g	gravity
h	heat transfer coefficient
k	heat conductivity
k_{ij}	ratio of i-layer to j-layer conductivity
L_i	i-layer height
L_{ij}	ratio of i-layer height to j-layer height
Nu	Nusselt number, $Nu = hL/k$
Pr	Prandtl number, $Pr = \nu/\alpha$
Q	heat transfer rate
Q_v	volumetric heat generation rate
q	heat flux
Ra	Rayleigh number, $Ra = g\beta Q_v L^5/\nu\alpha k$
Ra_2	Rayleigh number based on the reference values of the lower layer
Ra_e	external Rayleigh number, $Ra_e = g\beta\Delta TL^3/\nu\alpha$
T	temperature
X	fluid layer length

Greek symbols

α	thermal diffusivity
α_{ij}	ratio of i-layer to j-layer thermal diffusivity
β	coefficient of thermal expansion
η	fraction of heat generated within the layer that is transferred downward
ν_{ij}	ratio of i-layer to j-layer kinematic viscosity
ν	kinematic viscosity

Subscripts

1	top layer
2	lower layer
exp	experimental
corr	correlation
int	interface
max	maximum
w	wall

Rayleigh number. However, these experimental correlations are valid only for the fluid systems and height ratios chosen in the tests, and are not universal. The uncertainty in the measured Nu and Ra is reported to be less than 5.2%. The numerical predictions in the vorticity-stream function formulation for a uniform computational domain of 28×40 were found to be in general agreement with the data. The interface was treated as a semisolid conducting sheet with interfacial shear transmitted from the lower layer to the upper layer. In addition, simulations were done with the interface assumed to be hydrodynamically rigid. It was concluded that for either interface, the overall flow pattern was not much affected by the hydrodynamics at the interface. However, predicted Nusselt number for the case of a hydrodynamically rigid interface was about 10% lower.

In this study I would like to suggest two simple semiempirical correlations for estimates of heat transfer in a two-layer system of immiscible fluids with the lower one heated from within. In contrast to the results of the previous studies, these correlations will show the dependence on the height ratio and fluid parameters.

2. Results and discussion

We consider an infinite horizontal layer of two superposed immiscible fluids with uniform internal heat generation in the lower layer, as shown in Fig. 1. The top boundary is kept at constant temperature T_w, and the bottom boundary is either insulated or isothermal at temperature T_w. The depth of the lower layer is L_2 and the depth of the top layer is L_1. The physical properties of the top and the bottom fluids are denoted by indices 1 and 2 respectively. We will treat the interface as a thin, highly thermally conductive, solid (non-slip) and stationary boundary. The interfacial forces due to surface tension variations produced by temperature gradients (Marangoni effects) are assumed to be negligible. We assume that thermal equilibrium has been reached, and that longitudinal variation of the temperature at the fluid–fluid interface, is negligible. Similar approach was used in the study of thermal convection in a layered system of two immiscible liquids heated from below in Ref. [9].

We would like to (a) estimate the dimensionless heat transfer coefficient as a function of appropriate parameters in case of adiabatic lower boundary; (b) estimate fraction of heat generated within the layer that is transferred downwards in case of isothermal bottom surface.

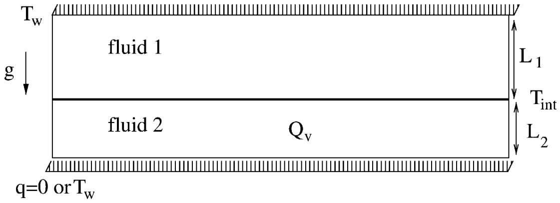

Fig. 1. Schematic diagram of the two-layer problem.

2.1. Adiabatic bottom surface

Here, we suggest a conservative semiempirical correlation to estimate dimensionless heat fluxes in the system described above. The expression obtained will be validated with the experimental data of Kulacki and Nguen and may be used for any immiscible fluid system and height ratio, L_{12}. The experimental results of Kulacki and Nguen [13] are summarized in Table 1. The thermophysical properties of heptane, silicone oil, and water are given in Table 2.

We assume that (a) the correlation of the following form is valid in the lower heated layer: $Nu_2 = C_2(Pr_2) \cdot Ra_2^{n_2}$. The constants $C_2(Pr_2)$ and n_2 can be taken from known heat transfer correlations for horizontal fluid layers with an adiabatic lower boundary and an isothermal upper wall (see Table 3); (b) the heat transfer in the upper non-heat generating layer is described by an expression of the type $Nu_1 = C_1(Pr_1) \cdot Ra_1^{n_1}$. The constants n_1 and C_1 can be taken from some known correlations for Rayleigh–Bénard convection (see Table 4).

We estimate the Nusselt number, defined as

$$Nu = \frac{q(L_1 + L_2)}{k_2 \cdot (T_{\max} - T_{\mathrm{w}})}, \tag{1}$$

where the heat flux through the interface is $q = Q_{\mathrm{v}} L_2$. The Rayleigh numbers Ra_1 and Ra_2 are defined as $Ra_1 = g\beta_1 L_1^3 \cdot (T_{\mathrm{int}} - T_{\mathrm{w}})/\alpha_1 \nu_1$ and $Ra_2 = g\beta_2 Q_{\mathrm{v}} L_2^5/\alpha_2 \nu_2 k_2$,

Table 1
The experimental Nusselt number correlations of Kulacki and Nguen [13]

Top layer	L_{12}	Nu	Range of Ra_2
Silicon oil	0.035	$0.186Ra_2^{0.1986}$	$10^7 < Ra_2 < 10^{11}$
Silicon oil	0.111	$0.183Ra_2^{0.1988}$	$10^7 < Ra_2 < 10^{11}$
Silicon oil	0.433	$0.115Ra_2^{0.2262}$	$10^4 < Ra_2 < 10^{11}$
Heptane	0.04	$0.126Ra_2^{0.225}$	$10^7 < Ra_2 < 10^{11}$
Heptane	0.111	$0.112Ra_2^{0.232}$	$10^7 < Ra_2 < 10^{11}$
Heptane	0.433	$0.135Ra_2^{0.232}$	$10^5 < Ra_2 < 10^{11}$

Table 2
Physical properties (taken at $T = 30$ °C)

Fluid	ρ (kg m^{-3})	β (K^{-1})	c_p (J kg^{-1} K^{-1})	k (W m^{-1} K^{-1})	μ (kg m^{-1} s^{-1})	Pr
Water	995	3×10^{-4}	4200	0.61	9×10^{-4}	6
Silicon oil	920	1.1×10^{-3}	1860	0.12	4.6×10^{-3}	72
Heptane	670	7.1×10^{-4}	2050	0.13	3.9×10^{-4}	6

Table 3
Experimental correlations for internally-heated horizontal fluid layers with an adiabatic lower wall and an isothermal upper boundary

References	Nu	Pr	Ra	L/X
[7]	$0.449Ra^{0.228}$	6–7	4×10^5–10^9	0.29–1.65
[12]	$0.258Ra^{0.239} \pm 5\%$	6.2–6.6	3×10^5–5×10^9	0.05–0.25
[10]	$0.338Ra^{0.227} \pm 5\%$	2.8–6.9	3.8×10^3–4.3×10^{12}	0.025–0.5

Table 4
Correlations for Rayleigh–Bénard convection

References	Nu	Pr	Ra
[8]	$0.069Ra^{1/3}Pr^{0.074}$	0.02–8750	1.5×10^5–6.8×10^8
[17]	$0.00238Ra^{0.816} \pm 4\%$		1.7×10^3–3.5×10^3
	$0.229Ra^{0.252} \pm 7\%$		3.5×10^3–10^5
	$0.104Ra^{0.305}Pr^{0.084} \pm 12\%$		10^5–10^9
[4]	$0.183Ra^{0.278}$	$Pr \sim 7$	2.76×10^5–1.05×10^8
[2]	$0.23 \pm 0.03Ra^{0.282\pm0.006}$	0.65–1.5	4×10^7–6×10^{12}
[3]	$0.140 \pm 0.005Ra^{0.26\pm0.02}$	$Pr \sim 0.025$	7×10^6–4.5×10^8
[14]	$0.124Ra^{0.309\pm0.0043}$	$Pr \sim 0.7$	10^6–10^{17}

respectively. Thus, the expressions for the Nusselt numbers in both layers can be written as

$$Nu_1 = \frac{Q_v L_2 L_1}{\kappa_1 (T_{int} - T_w)} = C_1 \cdot \left(g \cdot \frac{L_1^3}{\alpha_1 \nu_1} \right)^{n_1} \cdot (\beta_1 (T_{int} - T_w))^{n_1},$$

$$Nu_2 = \frac{Q_v L_2^2}{k_2 (T_{max} - T_{int})} = C_2 \cdot \left(g \cdot \frac{L_2^3}{\alpha_2 \nu_2} \right)^{n_2} \cdot \left(Q_v L_2^2 \cdot \frac{\beta_2}{k_2} \right)^{n_2}. \quad (2)$$

Adding $(T_{int} - T_w)$ to $(T_{max} - T_{int})$, and taking into account Eq. (1), we obtain the following particular simple expression for the Nusselt number:

$$Nu = \frac{(1 + L_{12}) \cdot C_2 \cdot Ra_2^{n_2}}{1 + A \cdot C_2 \cdot Ra_2^{(-n_1 + n_1 n_2 + n_2)/(n_1+1)}}, \quad (3)$$

where $A = [(\alpha_{12}\nu_{12})^{n_1}/C_1 \beta_{12}^{n_1} k_{12} L_{12}^{3n_1-1}]^{1/(n_1+1)}$. The Nusselt number depends on the dimensionless parameters Ra_2, L_{12} and A. It should be noted that the underlaying correlation for the lower layer is based on the experimental data on water, thus, C_2 does not depend on the Prandtl number, Pr_2. In general, $Nu = f(Ra_2, Pr_2, L_{12}, A)$.

The Nusselt number as a function of Ra_2 for heptane–water and silicon oil–water is plotted in Figs. 2 and 3. The Nusselt number values given by the correlation developed in the present work are denoted by dashed lines and those given by the correlation of Kulacki and Nguen are denoted by solid lines. The constants are taken from O'Toole and Silveston [17] for the upper layer, $C_1 = 0.101 Pr_1^{0.084}$ and $n_1 = 0.333$, and the Kulacki and Emara [10] correlation for the lower layer, $C_2 = 0.338$ and $n_2 = 0.227$. The O'Toole and Silveston correlation is chosen primarily for its application to a wide range of the Prandtl number. The constants are chosen for the turbulent regime. The application of other correlations (e.g., Globe and Dropkin [8]) does not change the results appreciably. It can be seen that the suggested estimate is consistent with correlated experimental data as a function of Ra_2. The underprediction (up to 30%) might be due to uncertainty of physical properties and experimental correlations used at individual fluid layers. The hydrodynamic coupling through the interface as well as effects of side boundaries neglected in the model can also affect heat transfer through the density–stratified interface.

It is interesting to note that since most of the correlations for the upper layer give the power exponent $n_1 \sim 1/3$, the heat transfer in the two-layer system does not depend strongly on L_{12}. In the limiting case when heat is transferred in the upper layer by means of conduction only, $C_1 = 1$, $n_1 = 0$, $Nu \sim 1/(Ra_2^{-n_2} C_2^{-1} + L_{12}/k_{12})$. In such a case, the dimensionless heat transfer coefficient in a double-layer configuration will be close to that in a one-layer fluid only when $L_{12} \to 0$ or $k_{12} \to \infty$, and decrease with the increase of L_{12} or the decrease of k_{12}.

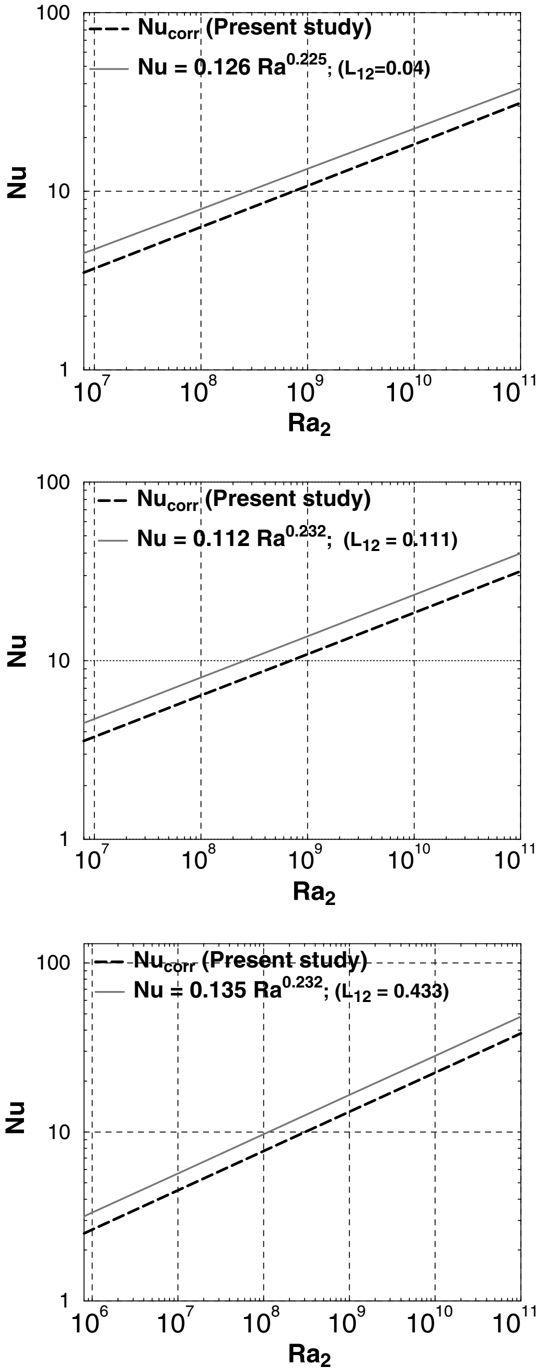

Fig. 2. Semiempirical (dashed lines) and experimental (solid lines) Nusselt number as a function of the Rayleigh number for heptane–water.

2.2. Isothermal bottom surface

We consider two immiscible density–stratified fluids, bounded at the top and the bottom by rigid walls and

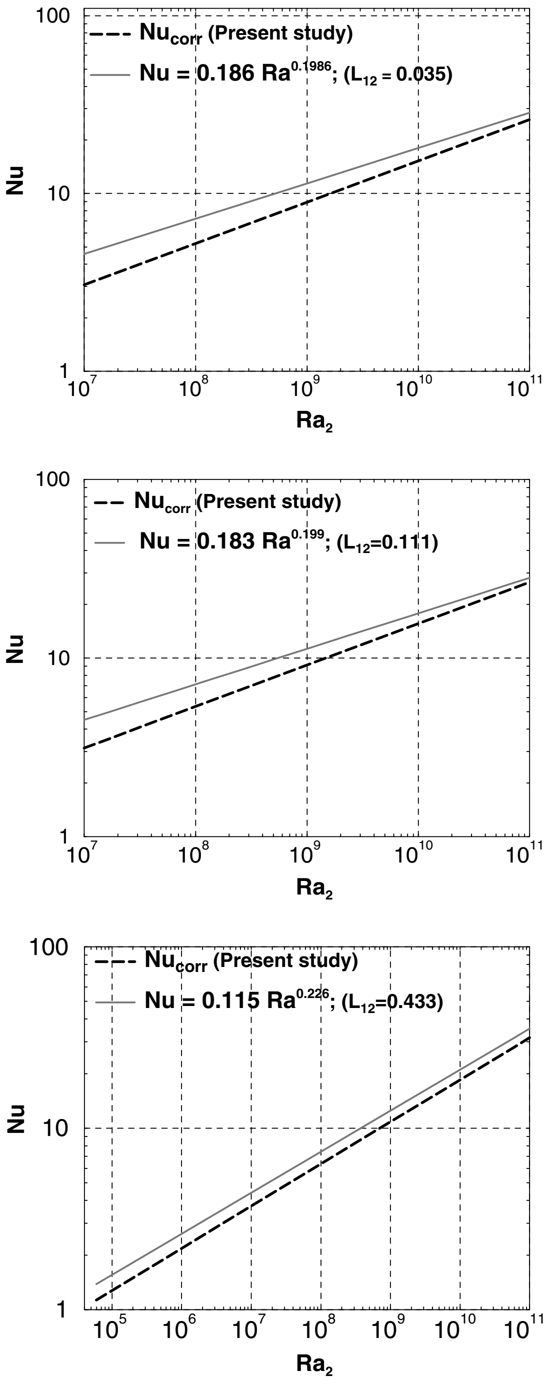

Fig. 3. Semiempirical (dashed lines) and experimental (solid lines) Nusselt number as a function of the Rayleigh number for silicon oil–water.

kept isothermal at an equal temperature T_w. The lower layer is heated by uniformly distributed heat sources. We would like to predict fraction of the heat generated within the layer that is transferred downwards, η.

It is assumed that (a) heat transfer in the lower layer can be estimated by the correlation of Baker et al. [1];

$$\frac{(1-\eta)^{0.865}}{\eta^2 - 2Ra_e/Ra_2} = \frac{1}{2}C_2 Ra_2^{n_2}, \tag{4}$$

where $Ra_e = g\beta_2 L_2^3 \cdot (T_{int} - T_w)/\alpha_2 \nu_2$. This correlation was developed to evaluate the upward and downward heat fluxes from an internally heated layer with unequal boundary temperatures. The correlation is based on the experimental data of Kulacki and Goldstein [11] for water. Constants C_2 and n_2 were taken from the correlation of Kulacki and Emara [10]; (b) As before, we assume that the heat transfer in the top layer can be correlated as $Nu_1 = C_1(Pr_1) \cdot Ra_1^{n_1}$, where $Ra_1 = gL_1^3\beta_1(T_{int} - T_w)/\alpha_1\nu_1$.

Defining

$$Nu_1 = \frac{Q_v(1-\eta)L_1L_2}{k_1(T_{int} - T_w)}, \tag{5}$$

and using the correlation for the upper layer, we express T_{int} as

$$T_{int} = T_w + \left[\frac{Ra_2(1-\eta)L_{12}\beta_{12}}{k_{12}\left(g\cdot\frac{L_2^3}{\alpha_2\nu_2}\right)\left(g\cdot\frac{L_1^3}{\alpha_1\nu_1}\right)^{n_1}C_1}\right]^{1/(n_1+1)} \cdot \beta_1^{-1}. \tag{6}$$

Introducing T_{int} into Ra_e, we obtain the following correlation for η:

$$\frac{(1-\eta)^{0.865}}{\eta^2 - 2A\frac{(1-\eta)^{1/(n_1+1)}}{Ra_2^{n_1/(n_1+1)}}} = \frac{1}{2}C_2 Ra_2^{n_2} \tag{7}$$

for $\eta \in (0, 1]$. Thus, we have obtained the equation for η as a function of the dimensionless groups Ra_2 and A, defined as before. Eq. (7) can be solved numerically. The values of η as a function of Ra_2 are plotted in Fig. 4. We

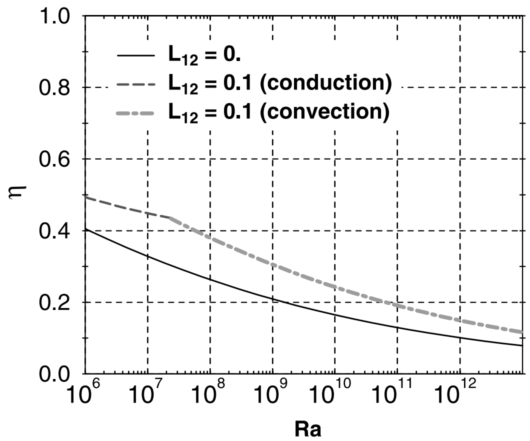

Fig. 4. Dependence of the fraction of heat transfer downwards on Ra_2. Heat is transferred in the upper layer by conduction only for $Ra^2 < 2 \times 10^7$ ($A = 1$, $Pr_1 = 4$).

took values for C_2 and n_2 from the correlation of Kulacki and Emara [10]. It can be seen that more heat is being transferred downwards in a double-layer configuration as the upper layer imposes additional thermal resistance. As Ra_2 increases, the difference in η between one- and two-layer cases decreases. Maximum possible values of η corresponding to the case of conduction heat transfer in the top layer are defined by the equation

$$\frac{(1-\eta)^{0.865}}{\eta^2 - 2(L_{12}/k_{12})(1-\eta)} = \frac{1}{2} C_2 Ra_2^{n_2}. \tag{8}$$

3. Summary

In this paper, natural convection heat transfer in a two-fluid stratified system with uniform internal heat sources is investigated theoretically. The case of cooled top surface in a rectangular cavity is considered. A general semiempirical correlation is suggested to estimate the heat transfer coefficient as a function of appropriate dimensionless parameters. The predicted results are compared with known experimental correlations. Unlike the previous correlations, the suggested correlation can be applied for fluids of different physical properties. For the case of top and bottom cooling, a correlation to estimate the fraction of heat generated within the lower layer that is transferred downwards is proposed. The range of the applicability of these correlations is defined by that of the underlying single-layer correlations.

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